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217 McHlaine hall

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taken out of 16

September 21, 2000

MATHEMATICS 110 (31)

Total Marks - 18

Quiz #1

Time: 45 minutes

Last Name: \_\_\_\_\_ Student Number: \_\_\_\_\_

[5]

1. SHORT ANSWER SECTION (Answers will be marked either RIGHT or WRONG.)

Evaluate the following EXACTLY:

a)  $\cos(\pi/3) = \frac{\pi}{3} = \frac{180^\circ}{3} = 60^\circ \quad \cos 60^\circ = \cos \frac{1}{2} \quad = \frac{1}{2}$

b)  $\tan(11\pi/4) = \frac{11(180^\circ)}{4} = 4 \frac{45^\circ}{175^\circ} \quad \frac{45^\circ}{175^\circ} \quad 180^\circ - 135^\circ = 45^\circ \quad \tan 45^\circ = \frac{1}{1} (-1)$

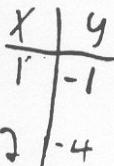
c)  $\sin(5\pi/12) = \frac{72^\circ}{90^\circ} \quad \frac{72^\circ}{90^\circ} \quad 75^\circ \quad \sin 45^\circ + \sin 30^\circ = \frac{1}{\sqrt{2}} + \frac{1}{2} \quad \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1+\sqrt{2}}{2\sqrt{2}}$

State whether the following statements are TRUE or FALSE:

d) The relation given by  $x + 2y - 6 = 0$  defines a function  $f$  with  $y = f(x)$  and domain  $(-\infty, \infty)$ .  $\frac{2y}{2} = \frac{6-x}{2} \quad y = \frac{-x}{2} + 3$

e) The relation given by  $x + 2y^2 - 6 = 0$  defines a function  $f$  with  $y = f(x)$  and domain  $(-\infty, 6)$ .  $\frac{2y^2}{2} = \frac{6-x}{2} \quad y = \sqrt{\frac{-x}{2} + 3}$

f) The relation given by  $x + 2\sqrt{y} - 6 = 0$  defines a function  $f$  with  $y = f(x)$  and domain  $(-\infty, 6)$ .  $2\sqrt{y} = 6-x \quad y = (12-2x)^2$



Find the following:

g) The slope of a line through the point  $(0, 1)$  and parallel to the line  $y = -3x + 2$ .  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-1}{0-2} = \frac{0}{-2} = -3$

h) The slope of a line through the points  $(0, 1)$  and  $(-1, 1)$ .

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-1}{0+1} = \frac{0}{1} = 0$  this is a horizontal line and the slope is 0

i) The domain of the function  $f(x) = x^3 - 3x^5 + 7 - x$ .

j) The domain of the function  $f(x) = \sin(\sqrt{x})$ .

[5]

2. Solve the following inequalities and express your answer using interval notation.

Show all your work.

a)  $-3 \leq 5 - 2x \leq 2$

$$-3 \leq 5 - 2x$$

$$0 \leq 2x - 5$$

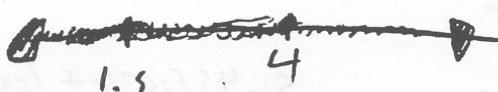
$$4 \leq x$$

$$-3 \leq 5 - 2x \quad 5 - 2x \leq 2$$

$$\frac{-8 \leq -2x}{-2} \quad \frac{5 \leq 2x}{-2}$$

$$4 \geq x \quad x \leq 2.5$$

$$x \geq 1$$



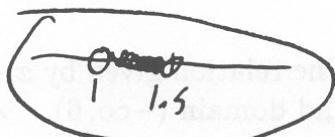
$$[1.5, 4]$$

b)  $\frac{x^2 + 1}{3 - 2x} < 2$

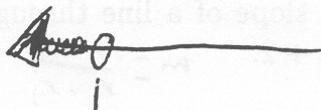
$$\frac{x^2 + 1}{3 - 2x} - 2 < 0$$

$$\frac{x^2 + 1 - 6 + 4x}{3 - 2x} < 0$$

$$\frac{x^2 + 4x - 5}{3 - 2x} < 0$$



$$\frac{(x-5)(x+1)}{3-2x} < 0$$



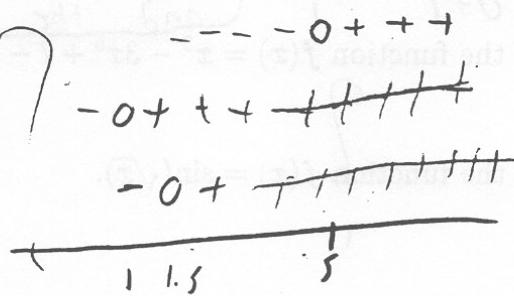
$$(-\infty, 1)$$

$$x - 5 < 0$$

$$x + 1 < 0$$

$$3 - 2x > 0$$

$$\frac{x^2 + 4x - 5}{3 - 2x} < 0$$



[4]

3. Consider the following function:

$$f(x) = 1 - |2 - x|$$

- a) Give a piecewise defined formula for  $f(x)$  which does not involve absolute values.

$$f(x) = \begin{cases} 2-x & \text{if } 2-x \geq 0 \\ -2+x & \text{if } 2-x < 0 \end{cases}$$

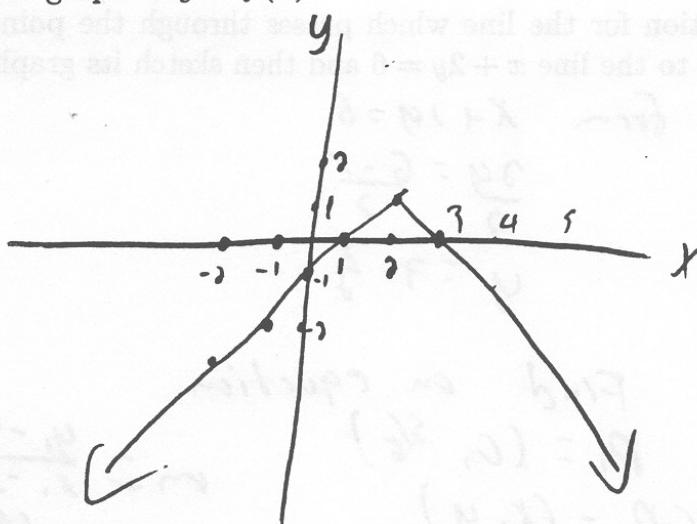
$$2-x \quad \text{if } x \geq 2$$

$$-2+x \quad \text{if } x < 2$$

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- b) Sketch the graph of  $y = f(x)$ .

$x$	$y$
-1	-2
0	-1
1	0
2	1
3	2
4	3
5	4



- c) Find the domain and range of  $f(x)$ .

located at graph

$$\{x \in \mathbb{R}\}$$

$$\{y \in \mathbb{R} \mid y \leq 1\}$$

- [2] 4. Prove or disprove that the function  $f(x) = -3(x-2)^2 + 6$  is an even function.

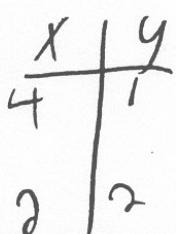
$$\begin{array}{ll} f(x) & f(-x) \\ \left\{ \begin{array}{l} (x-2)^2 + 6 \\ (x^2 - 4x + 4) + 6 \\ 3x^2 + 12x - 12 + 6 \\ 3x^2 + 12x - 6 \end{array} \right. & \left. \begin{array}{l} -3(-x-2)^2 + 6 \\ -3(x^2 + 4x + 4) + 6 \\ -3x^2 - 12x - 12 + 6 \\ -3x^2 - 12x - 6 \end{array} \right. \end{array}$$

LHS  $\neq$  RHS

C. This is not an even function

- [2] 5. Find an equation for the line which passes through the point  $(0, 5/6)$  and is perpendicular to the line  $x + 2y = 6$  and then sketch its graph.

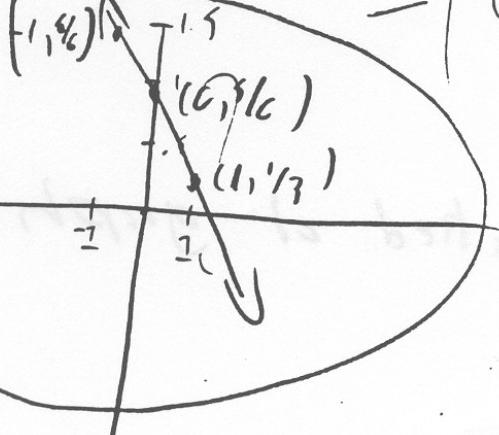
find 2 points from  $x + 2y = 6$



$$\begin{aligned} 2y &= 6 - x \\ y &= 3 - \frac{x}{2} \end{aligned}$$

$$\begin{aligned} m &= \frac{y_1 - y_2}{x_1 - x_2} \\ &= \frac{1 - 2}{4 - 2} \\ &= \frac{-1}{2} \end{aligned}$$

$$m = \frac{-1}{2}$$



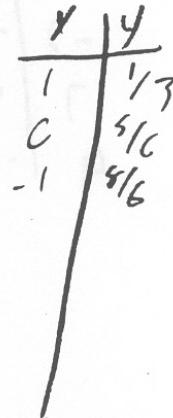
$$\begin{aligned} \text{Find an equation} \\ P_1 &= (0, \frac{5}{6}) \\ GP &= (x, y) \\ m &= \frac{y_1 - y_2}{x_1 - x_2} \end{aligned}$$

$$m = \frac{1}{2}$$

$$\begin{aligned} m &= \frac{y_1 - y_2}{x_1 - x_2} \\ &= \frac{1}{2} = \frac{y - \frac{5}{6}}{x - 0} \end{aligned}$$

$$-x = 2y - \frac{10}{6}$$

$$\begin{aligned} \frac{10}{6} - x &= 2y \\ 2 & \end{aligned}$$



$$\boxed{\frac{5}{6} - \frac{x}{2} = y}$$

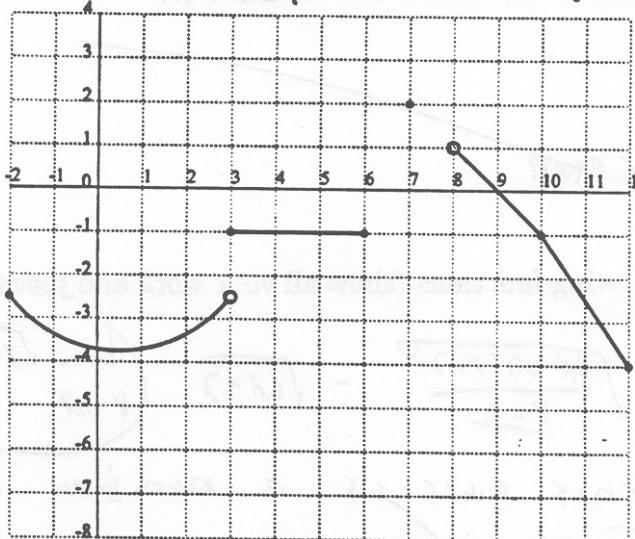
October 5, 2000

MATHEMATICS 110 (31)  
Midterm #1Total Marks - 35  
Time: 80 minutes

Last Name: \_\_\_\_\_ Student Number: \_\_\_\_\_

8 [10]

1. Consider the following graph of the function  $f$  on the interval  $[-2, 12]$ :



- a) Assuming that the domain of  $f$  is a subset of  $[-2, 12]$ , give the domain and range of  $f$ ?  $D_f = [-2, 6] \cup [7, 12]$

$$R_f = [-4, 1] \cup 2$$

- b) Give values (if possible) for the following or state that it does not exist.

$$\lim_{x \rightarrow -2^+} f(x) = -2.5$$

$$\lim_{x \rightarrow 7} f(x) = DNE$$

$$\lim_{x \rightarrow 3^-} f(x) = -2.5$$

$$\lim_{x \rightarrow 8^+} f(x) = 1$$

$$\lim_{x \rightarrow 4} (f(x) + 1)^2 = 0$$

$$\lim_{x \rightarrow 6^-} f(x) = -1$$

$$\lim_{x \rightarrow 10} f(x) = -1$$

$$f(8) = DNE$$

$$f(7) = 2$$

$$f(10) = -1$$

- c) Determine all points  $a$  in the domain of  $f$  where  $f$  is discontinuous and justify your answer by stating which continuity condition fails. State the interval(s) in the domain of  $f$  such that  $f$  is continuous on each interval and such that each interval is as large as possible.

pts

3, 6, 7, 8 (1) all are in domain

Because  $f(x)$   
6, 7, 8

[2]

2. Consider

$$F(x) = \sqrt{\frac{1}{e^x + 1}}$$

a) Find functions  $f$ ,  $g$ , and  $h$  such that  $F(x) = f(g(h(x)))$ 

-2

b) What is the domain of  $F(x)$ ?

[6]

3. Find the domain of the following functions (show all your work and justify your answer):

$$\text{a) } f(x) = \sqrt{\frac{x^2 - 4}{x - 2}} = \sqrt{\frac{(x-2)(x+2)}{x-2}} = \sqrt{x+2}$$

$$\lim_{x \rightarrow 2} \sqrt{x+2} = \infty$$

-  $x \neq 2$  because this gives us a number over 0 which is undefined.

- Because the  $\sqrt{ }$  is already in the question we only take the positives.

-1.5

$$\therefore D_f = (2, \infty)$$

$$\text{b) } f(x) = \frac{1}{\sqrt{6-x-x^2}}$$

$$\lim_{x \rightarrow 2} f(x)$$

$$\lim_{x \rightarrow 2} \frac{1}{\sqrt{6-x-x^2}} =$$

$$(-) 2^+$$

$$\therefore D_f = (-\infty, 2)$$

$$\lim_{x \rightarrow 2} \frac{1}{\sqrt{6-x-x^2}} = -\infty$$

-3

$$\begin{array}{r} 1.9 \\ 1.9 \\ \hline 17.1 \\ 14.6 \\ \hline 2.51 \end{array}$$

$$2.71 \quad 6 - 2.71 = 3.69$$

$$1.71 \quad (-1.71) = 4.29$$

$x = 2$

4. Find all vertical asymptotes of  $f(x) = \frac{x^2+x-6}{x^2-9}$  and justify your answer by finding all relevant limits.

$(x+3)(x-3)$  only possibilities at 3, ✓

$$\lim_{x \rightarrow 3^+} f(x)$$

$$\lim_{x \rightarrow 3^-} f(x)$$

[10]

5. Evaluate the following limits.

$$\text{a) } \lim_{x \rightarrow -3^+} \frac{x^2 + 2x - 4}{(x+2)^3} = \frac{-1,1}{-1,1}$$

20  
11

$$= 1 \quad \checkmark$$

X	Y
-3.9	+1.1
-3.49	-1.01
-3.999	+1.001
-3.9999	+1.0001
-3	1

$$\text{b) } \lim_{x \rightarrow -2^-} \frac{x^2 + 2x - 4}{(x+2)^3} = \frac{-4}{-0.001} = 4000 \quad = \infty \quad \checkmark$$

X	Y
-2.1	4000
-2.01	4000
-2.001	
-2.0	DNE asymptote

$$\text{c) } \lim_{x \rightarrow -1} \frac{2x+2}{x^2 - x - 2}$$

asymptote at  $x = -1$  DNE

$$\frac{2(-0.9)+2}{(-0.9)^2 + (-0.9) - 2} = \frac{-1.8+2}{0.81 + 0.9 - 2} = \frac{0.2}{-0.29} = \frac{-0.2}{0.29}$$

X	Y
-1	DNE
-1.1	1
-0.9	0.2

Approaching diff # No Limit or DNE

$$\text{d) } \lim_{t \rightarrow 3} \frac{\sqrt{t+6} - 3}{3t^2 - 27}$$

$$= \frac{\sqrt{a+3}}{3(3)^2 - 27} = \boxed{0}$$

$$9^2$$

$$9^2$$

$$9^2$$

5. Consider the following piecewise defined function:

$$g(x) = \begin{cases} x^2 - 2 & x \leq 3 \\ x & x > 3 \end{cases}$$

a) Prove that  $g$  is discontinuous at  $x = 3$ .

$$\lim_{x \rightarrow 3^-} g(x) = g(a)$$

$x \rightarrow a^-$

$$\lim_{x \rightarrow 3^-} x^2 - 2 = 7$$

$x \rightarrow 3^-$

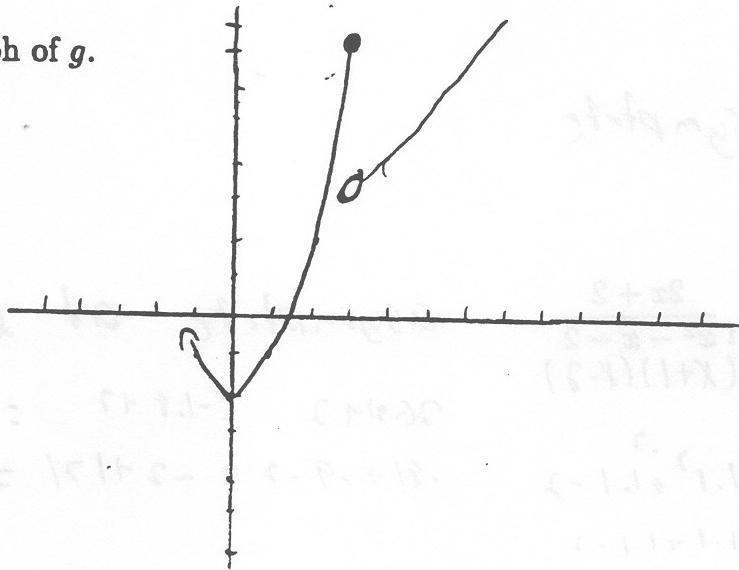
$$\lim_{x \rightarrow 3^+} y = 3$$

③  $g(a) = 7 \neq 3$  so it is not continuous

(3 is in domain)

$x \leq 3$

b) Sketch a graph of  $g$ .



c) Consider the following piecewise defined function:

$$h(x) = \begin{cases} x^2 - 2 & x \leq 3 \\ x - k & x > 3 \end{cases}$$

For what value(s) of  $k$  is  $h(x)$  continuous at  $x = 3$ ? Justify your answer.

$$h(x) = x - H$$

$$7 = 3 - H$$

$$h(x) = 7 \leftarrow 3^2 - 2 = 7$$

$$x = 3$$

